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Altitude-Velocity Charts for Imperfect Air

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NUMEROUS authors have computed normal shock-wave parameters based on perfect gas‡ thermodynamic properties and various "standard" atmospheres (e.g., see Feldman,[1] Wittliff and Curtis,[2] and Marrone[3]). At sufficiently low pressures, except for changes in standard atmosphere conditions, those results should be sufficiently accurate for most engineering purposes. However, at high pressures (or more correctly at high densities), the effects of intermolecular forces can become important and the thermodynamic properties are affected.

Lewis and Burgess[4] recently computed freestream stagnation conditions, conditions behind a normal shock, and nor-

Table 1 Ranges of conditions considered in altitude-velocity calculations

Altitude-velocity ranges			
Model atmosphere		Altitude range, kft	Velocity range, kft/sec
Wittliff and Curtis	1959	0–300	2–26
Marrone	1959	0–300	26–50
Lewis and Burgess	1962	10–400	6–50
Gasdynamic properties[a]			
Wittliff and Curtis[b] and Marrone[b]:	ρ_s/ρ_1, p_s/p_1, T_s/T_1, Z_s, H_s/H_1, H_0'/H_1, p_0'/p_1, T_0'/T_1, $\gamma_e{}^c$		
Lewis and Burgess:	H_0/H_1, p_0/p_1, ρ_0/ρ_1, T_0/T_1, a_0/a_a, Z_0, $\gamma_{E,0s}{}^d$; H_s/H_1, p_s/p_1, ρ_s/ρ_1, T_s/T_1, S_s/R, Re_1/ft, $\gamma_{E,s}$; $\dot q(r_n)^{1/2}$, p_0'/p_1, ρ_0'/ρ_1, T_0'/T_1, a_0'/a_a, Z_0', $\gamma_{E,0'}$		

[a] Subscript 1 denotes freestream, s behind normal shock, 0 freestream stagnation, 0′ normal shock stagnation, and a at 273.15°K and 1 atm.
[b] Gas composition behind the normal shock also given based on 14 specie model.
[c] $\gamma_e = 2M_1^{-2}(\rho_1/\rho_s - 1)^{-1} - 1$.
[d] $\gamma_E = (\partial \ln p/\partial \ln \rho)_S$.

Received December 10, 1964. This work was sponsored by the Arnold Engineering Development Center (AEDC), Air Force Systems Command, U. S. Air Force, under Contract No. AF 40(600)-1000 with ARO, Inc., Operating Contractor, AEDC.

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‡ The ideal gas obeys $p = \rho RT$, $h = C_pT$, and $\gamma = C_p/C_v =$ const. A perfect gas will denote one obeying $p = Z\rho RT$, which includes dissociation and ionization neglecting intermolecular effects. An imperfect gas obeys $p = Z\rho RT$, which includes dissociation, ionization, and intermolecular forces. Local thermodynamic [i.e., thermal, mechanical (pressure), and chemical] equilibrium is assumed to exist for all conditions.

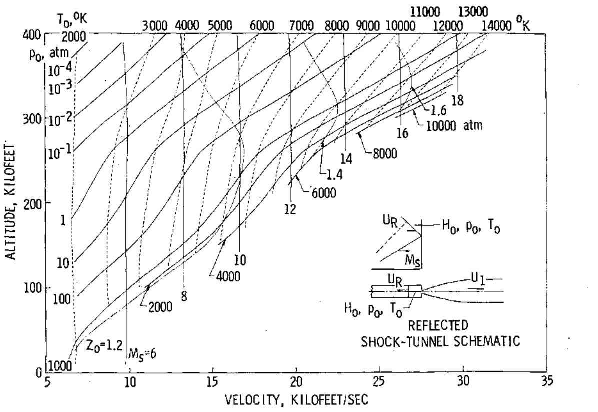

Fig. 1 Equilibrium freestream stagnation conditions necessary for flight duplication.

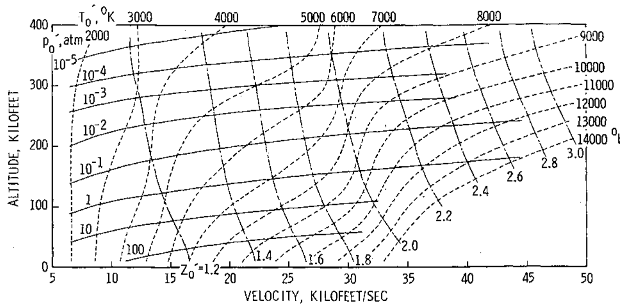

Fig. 2 Normal shock stagnation conditions at flight duplication.

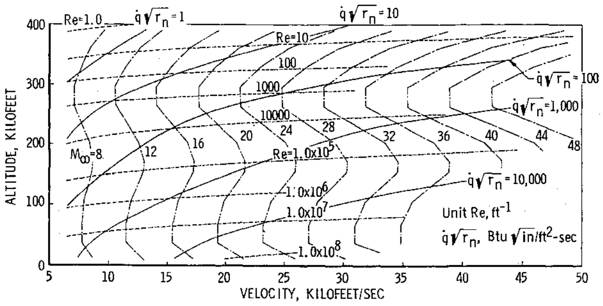

Fig. 3 Freestream Mach number, unit Reynolds number, and Fay-Riddell stagnation heat-transfer rate at flight duplication.

Table 2 Comparison of Air Research and Development Command (1959) and U S. Standard (1962) model atmospheres[a]

Geometric alt., kft	p_1	ρ_1	T_1	H_1/R^b	$a_1{}^b$
0	...	...	...	−0.4	...
70	−0.06	0.52	−0.57	−1.06	−0.32
100	−0.8	−3.16	2.50	2.00	1.21
120	2.44	−1.58	4.12	3.62	2.00
160	9.41	4.77	4.45	5.98	2.15
170	11.2	6.45	4.34	3.98	2.15
200	14.1	16.09	−1.75	−2.22	−0.92
220	11.15	15.5	−3.76	−4.21	−1.93
250	2.74	10.2	−6.74	−7.16	−3.45
280	−11.25	−2.96	−8.29	−8.70	−4.27

[a] Data shown were computed from percent difference = 100 [x(1959) − x(1962)]/x(1962).
[b] H(1959) and a(1959) were obtained from Wittliff and Curtis.[2]

Table 3 Comparison of present with previous perfect gas results[a]

Altitude, kft →	70	100	120	170	200	220	250	280
Velocity × 10^{-3} →	36	26	40	35	26	28	45	32
M_1	0.33	−1.19	−1.95	−2.13	0.92	1.97	3.57	4.46
ρ_s/ρ_1	−0.79	0.24	−0.86	−0.49	−0.56	−0.49	−0.73	0.14
p_s/p_1	0.44	−2.47	−4.03	−4.35	1.70	3.79	7.10	8.98
T_s/T_1	1.38	−2.64	−3.41	−3.75	2.62	4.56	7.93	8.96
H_s/H_1	1.03	−1.99	−3.53	−3.85	2.17	4.33	7.64	9.46
H_0/H_1	1.09	−1.96	−3.47	−3.83	2.23	4.38	7.66	9.50
p_0'/p_1	0.44	−2.52	−4.07	−4.35	1.67	11.6	7.12	8.92
T_0'/T_1	1.34	−2.54	−3.35	−3.71	2.59	8.83	7.97	8.97

[a] Data shown were computed from percent difference = 100 [x(Marrone) − x(present results)]/x(present results).

mal shock stagnation conditions. The data were presented in graphical and tabular form over the ranges of alt = 10^4-(10^3)4 × 10^5 ft and velocity = 6000(1000)50,000 fps. The calculations were based on the imperfect air thermodynamic properties of Hilsenrath and Klein[5] and Lewis and Neel[6] and the 1962 U. S. Standard Atmosphere.[7]

Some of the results of the calculations are shown on Figs. 1–3. The sphere stagnation heat-transfer rates were computed from the Fay and Riddell formula[8] in the form used previously by Lewis and Burgess[9]:

$$\dot{q}(r_n)^{1/2} = 7.55274 \times 10^{-3}(\rho_w\mu_w)^{0.1}(\rho_0'\mu_0')^{0.4}(H_0 - H_w)(p_0'/\rho_0')^{0.25}$$

in Btu(in.)$^{1/2}$/ft^2-sec. The viscosity data were obtained from Hansen;[10] Lewis number was assumed unity, the sphere radius is in inches, and w and $0'$ denote wall (T_w = 300°K) and normal shock stagnation conditions.

Table 1 shows the ranges of conditions considered in the imperfect gas calculations of Lewis and Burgess and two previous perfect gas calculations of Wittliff and Curtis[2] and Marrone.[3] As noted in the table, the freestream stagnation conditions were not presented in the earlier perfect gas results, and it is these data that are most affected by imperfect gas effects.

A comparison was made between the 1959 Air Research and Development Command (ARDC) model[11] and the 1962 standard atmosphere used in the present work, and the results are shown in Table 2. The differences near 220,000 ft are as large as 15% and affect the normal shock solutions.

A comparison was made between the present results and those of Marrone[3] based on the 1959 model atmosphere. The results of that comparison are shown in Table 3. The differences shown in Table 3 are mainly due to differences in the standard atmospheres and are not due to imperfect gas effects since the densities are too low for these effects to be important. The freestream stagnation conditions shown on Fig. 1 above about 100 atm are, however, affected by imperfect gas effects.

The present results are based on the most recent thermodynamic data and model atmosphere known to the authors. Differences of about 10% were indicated in normal shock wave properties when compared with previous perfect gas results. These results are considered significant and the tabulated data in Ref. 4 are designed to permit easy interpolation.

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Vibration of Hub-Pin Plates

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PREVIOUS investigations have presented experimental data for the mode shapes and frequencies of rectangular, triangular, and skewed cantilever plates.[1,2] Recent experimental studies have confirmed the results of previous tests on cantilevered plates and also provided vibration data for hub-pin plates. The frequencies and nodal patterns of a square cantilever plate, a square hub-pin plate, a 45° cantilever plate, and a 45° hub-pin plate are shown in Fig. 1. A dimensionless frequency parameter Ω is given, where

$\Omega = 2\pi f/(D/\rho h a^4)^{1/2}$
f = frequency in cps
D = plate stiffness = $Eh^3/12(1 - \nu^2)$
ρ = density
h = thickness
a = length of root chord

Detailed deflection data for the hub-pin plates vibrating in the indicated modes are available in Ref. 4.

The frequencies and mode shapes were obtained by use of an electromagnetic shaker and a capacitance deflection probe shown in Fig. 2. The rotational restraint pin and a

Received December 8, 1964. Supported by the Navy Bureau of Weapons as a part of Contract No. NOrd-16498.

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